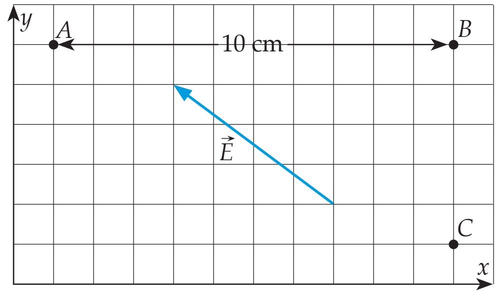
The outlined problems are examples of B-problems designed to work as classroom activities. These particular problems help students develop the line-integral concept.

E3B.8 Consider a region of space where the electric field is everywhere equal to $\vec{E} = [-400, +300, 0]$ N/C.



We'd like to calculate the potential difference $\phi_B - \phi_A$ between points A and B. We will do this by dividing the straight-line path going from A to B into 20 equal steps, which we will consider to be "sufficiently small" steps.

(a) List the components of $d\vec{r}$ for each step.

(b) Argue that $\vec{E}\cdot d\vec{r}$ has the same value for each step and compute that value. (*Hint:* Use the component expression for the dot product.)

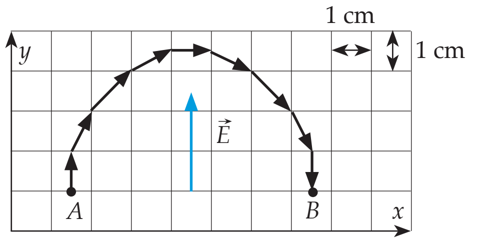
(c) Calculate the value of $\int \vec{E}\cdot d\vec{r}$ in this case.

(d) Calculate the potential difference $\phi_B - \phi_A$. If the potential at A is -5 V, what is the potential at B?

(e) In this particular case, show that $\int \vec{E}\cdot d\vec{r} = \vec{E}\cdot\ \vec{r}$, where $\Delta\vec{r}$ is the entire displacement from A to B? What would have to different about the electric field to make $\int \vec{E}\cdot d\vec{r} \neq \vec{E}\cdot\ \vec{r}$?

E3B.9 Consider the situation described in problem E3B.8. Repeat all of the parts of that problem for the path from B to C instead of A to B. (That is, the statement of that problem, replace "A" by "B" and "B" by "C" everywhere.)

E3B.10 Consider a region of space where the electric field is everywhere equal to $\vec{E} = [0, +200, 0]$ N/C. The spacing between grid lines in the diagram below is 1.0 cm.



We'd like to calculate the potential difference $\phi_B - \phi_A$ between points A and B along the curvy path shown in the diagram. I have divided this path into 9 steps, each of which we will consider to be "sufficiently small."

(a) List the components of $d\vec{r}$ for each step.

(b) Calculate the value of $\vec{E}\cdot d\vec{r}$ for each step. (*Hint:* Use the component expression $\vec{u}\cdot\vec{w} \equiv u_x w_x + u_y w_y + u_z w_z$.)

(c) Calculate the value of $\int \vec{E}\cdot d\vec{r}$ in this case.

(d) Calculate the potential difference $\phi_B - \phi_A$.

(e) Calculate $\int \vec{E}\cdot d\vec{r}$ and $\phi_B - \phi_A$ for a path going directly from A to B (dividing this path into six equal 1.0-cm steps). Is your result the same (or close to the same)?

E3B.11 Imagine that near a certain point in an equipotential diagram of the xy plane, the equipotentials are lines nearly parallel to the y axis 2 mm apart. The potential increases by 0.1 V per equipotential as one moves to the left. What is the magnitude and direction of the electric field in this region?

E3B.12 Imagine that in the neighborhood of a point P the electric field points in the $+y$ direction and has a magnitude of 1000 N/C. In the neighborhood of this point in the xy plane, argue that the equipotential lines will be parallel to the x axis and determine their spacing if there is a line for every 1-V potential difference.

E3B.13 Consider a particle with charge $Q = +2.78$ nC. The equipotential surfaces for such a particle will be spheres centered on the particle. On a sheet of paper in landscape orientation, draw a horizontal line, and then draw a dot near the paper's left edge to represent the particle. Then draw short vertical lines on the horizontal line that represent the points where the ten equipotential surfaces corresponding to $\phi = 100$ V, 200 V, …, 900 V, 1000 V intersect that radial line. Assume that $\phi \equiv 0$ at infinity.

E3B.14 The formula for the potential field of an infinite line with charge per unit length λ (see problem E3D.5) is

$$\phi = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_0}{r}\right) \tag{E3.26}$$

where r is the distance between point P where we are evaluating ϕ and the nearest point on the line and r_0 is the reference distance from the line where we set $\phi \equiv 0$. As we are free to choose r_0, let's arbitrarily set $r_0 = 2.0$ cm.

(a) Note that because the logarithm is unitless, $\lambda/2\pi\varepsilon_0$ must have the same units as ϕ. Assume that $\lambda/2\pi\varepsilon_0 \equiv B = 100$ V. Find the value of λ that yields this result.

(b) Using a ruler, construct a quantitatively accurate equipotential diagram for the potential field of this line, drawing equipotentials on both sides of the line for the range -150 V to 100 V in steps of 25 V.

E3B.15 Assume that the electric potential field in a certain region is $\phi(x, y, z) = ay$, where a is a constant. Compute the electric field vector $\vec{E}$ in this region.

E3B.16 Assume that the electric potential field in a certain region is given by $\phi(x, y, z) = a(x^2 + y^2 + z^2)$, where a is a constant. Find the electric field $\vec{E}(x, y, z)$ in this region.

Derivations

E3D.1 Consider a thin ring of radius R and total charge Q. Define a coordinate system so that the ring lies on the xy plane and is centered on the origin. Argue that the ring's potential field at an arbitrary point P on the z axis (which corresponds to the ring's central axis) is given by

$$\phi(z) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\sqrt{R^2 + z^2}} \tag{E3.27}$$

Explain your reasoning. (*Hint:* No calculus necessary!)