LAB 5 UNCERTAINTY OF THE MEAN

EXPERIMENTAL GOAL

We often encounter situations in the laboratory where we want to measure a quantity's value, but find that random measurement errors disguise the quantity's true value (as in the stopwatch measurements we did for the inertial mass lab). In such cases, we can get a *better* estimate of the quantity's "true value" by computing the *mean* (that is, the *average*) of a large number of measurements, because random errors (which are just as likely to lead to measured values that are too low as too high) will tend to cancel when we sum the measurement values to compute the mean.

"Better" implies that we are confident that the mean of a set of N measurements will be *closer* to the quantity's true value than any typical single measurement in that set is likely to be. This in turn implies that the standard deviation s_m that we would assign to a mean should be *smaller* than the standard deviation we would assign to a typical specific measurement. Moreover, we would expect s_m to decrease in comparison to s as N increases, because the greater the number of measurements that go into the mean, the more completely the random errors should cancel.

A plausible guess is that $s_m = s/N^b$, where b is some positive number (note that s_m in this formula decreases as N *in*creases and that $s_m = s$ as expected when $N = 1$.) Your goal in this lab is to check this hypothesis and determine b. You will do this using a computer program called *UncMean*. This program simulates a situation where hypothetical 20 lab teams all make N measurements of a quantity whose true value is 50 (in suitable units) but which are randomly perturbed so that the measurements each team gets fall into a Gaussian distribution centered on 50. The program displays the mean, and standard deviation s found by each team for their particular set of measurements, a histogram of any selected team's results that also shows the 20 team means, and (at the bottom) the standard deviation s_m of the 20 teams' means (computed in the usual way).

Find a way to use this information to plot a graph that will be linear if the hypothesis is true, and use your graph to determine the value of b. In formulas like this, b is usually an integer or a simple rational fraction. What does your data suggest is the likely value of b?

Assuming the hypothesis is true, then the *uncertainty* we would want to assign to the mean of a set of N measurements from a Gaussian distribution would be $U_m = ts_m = ts/N^b$, where t is the Student t factor for N. After you have determined a likely value for b, analyze the class data from the first two labs of the semester (see the blackboard) to determine (1) the uncertainty of the whole class' mean for the ratio between the inertial masses of iron and wood, and (2) whether the differences in the *mean* throwing distances of the six trebuchets are statistically significant.

LABORATORY SKILLS you will be developing

The main educational goal of this lab is to teach you the crucial concept of the uncertainty of the mean and its distinction from the uncertainty of a single measurement. Understanding this distinction will be essential for future labs. You will also practice graphing a non-linear relationship and some general uncertainty analysis skills.

SOME PROCEDURAL SUGGESTIONS AND NOTES

First think about what you are going to plot against what to get a straight-line graph, and the range of your independent variable. How large a value of N can the program handle in practice? Then think about how you are going to assign uncertainties to points on this graph. Note that if you press the "Calculate Means" Button several times, you get different values at the bottom. Why is this? This means that the values you plot are going to be somewhat uncertain. Figure out a way to handle this. If you are clever, you may be able to find a way of plotting things so that your independent variable has *zero* uncertainty.) Be *sure* that you check your procedure with your helper before taking a lot of data.

Your grader will ask you questions about the distinction between repeatable and unrepeatable measurements, and when you should use the uncertainty of a datum and when you use the uncertainty of the mean. *It is very important that you understand this.*