

“Will not your mind misgive you, when you find yourself in this gloomy chamber, too lofty and extensive for you, with only the feeble rays of a single lamp to take in its size...?”

--- Northanger Abbey

6.1 PRELIMINARY COMMENTS

The purpose of this chapter and the next is to provide some background information about the optics of thin lenses that you need for the optics labs and the *Speed of Light* lab. We are providing instruction in optics in this course in context of the laboratory because studies have suggested that this is a topic that students learn much better from laboratory experiences than from classroom instruction.

6.2 THE RAY MODEL OF LIGHT

The behavior of light in most optical systems whose components are larger than few a tenths of a millimeter can be most easily understood in terms of a *particle* model of light. (We will discuss various models of light and the reason for the particle model's size limitation at the end of Unit *E* and the first few chapters of Unit *Q*). The particle model of light imagines that light consists of infinitesimally tiny particles that we call **photons**, which travel in straight lines through a vacuum, (essentially) straight lines through air (or any other uniform medium), and either rebound from or are absorbed by solid opaque objects. We call a possible photon trajectory through an optical system a **ray** of light and can represent it on a diagram of the system by a thin line.

Photons (say from the sun) falling on any point on the surface of an opaque object are typically scattered in all possible directions. If the surface is particularly shiny or glossy, there may be more photons scattered in some directions than others, but usually there are at least some photons scattered in any given direction. Since light travels in straight lines in air, the rays representing the possible paths of photons scattered from such a point therefore look like straight lines pointing radially away from the point as shown in Figure 6.1. When we look at such an object, our eyes collect some of the scattered photons and our brains construct a mental image of the object based on the number, color, and trajectory of the photons collected.

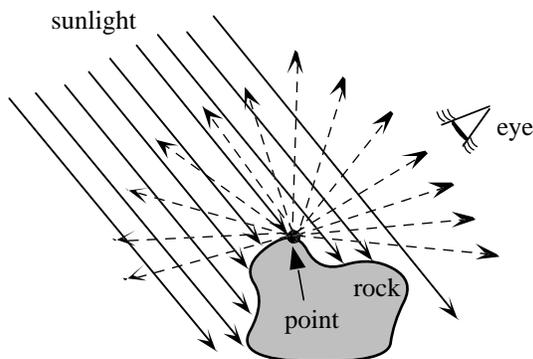


Figure 6.1: An illustration of how we can represent photon paths on a diagram using rays. The solid rays on this diagram represent paths of photons coming from the sun, while the dashed lines represent some possible paths for photons scattered from a specific point on a rock. (The rays indicate only the possible *directions* that photons can travel: the *length* of a ray on a diagram, unlike a vector, has no particular physical meaning.)

Note that the light that scatters from a given point on an illuminated object radiates away from the point in much the same way that it would if we were to replace the scattering point with a point *source* of light. In ray diagrams, we will typically ignore the light illuminating an object and treat points on the object as if they *emit* light.

Ray diagrams also typically illustrate the rays emanating from only one or two points on an object, and even then just a handful of possible photon trajectories radiating from those points. You should always keep in mind that *every* point on an opaque object radially scatters light in an infinite number of possible directions.

6.3 THE FOCAL POINT OF A CONVERGING (CONVEX) LENS

A **lens** is a transparent object that bends the trajectories of photons moving through it. (We will discuss why materials like glass bend light in the third chapter of Unit *Q*). A **converging lens** is constructed with the just right shape so that it bends parallel rays of light to a single point, which we call the **focal point** of the lens, as shown in Figure 6.2. We call the distance between the center of the lens and the focal point the lens' **focal length** f . Note that such a lens actually has two focal points: parallel rays going through the lens from the left will converge at the focal point on the right, while parallel rays coming from the right would converge at the focal point on the left (such rays are not shown in Figure 6.2). Both these focal points lie on a line that goes through the lens perpendicular to the plane of the lens: we call this line the **optic axis** of the lens.

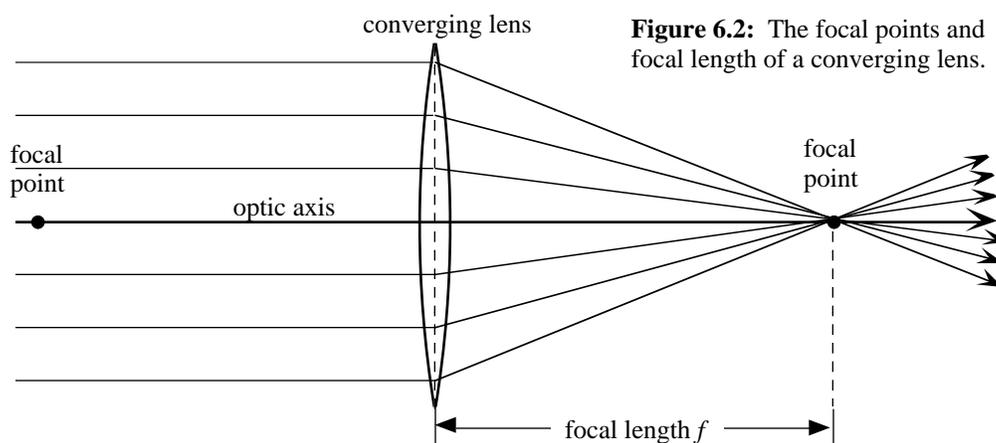


Figure 6.2: The focal points and focal length of a converging lens.

Note that in Figure 6.2, we have drawn the rays as if the lens bends them suddenly at the exact center of the lens. Actually, each ray is bent a certain amount when it enters the lens and a certain amount more when it leaves the lens. If the lens is very thin compared to its focal length (or other important distances in an optical system) details like this can be ignored and we can use the simple model that each ray is bent at the center. If a lens satisfies this criterion in a given context, we call it a **thin lens**.

It turns out that a thin lens will be a converging lens in the sense described above if its two surfaces are both small patches of a spherical surface (and the lens is wider at the center than at the edges). In the *Lens Systems* experiment later in the semester, you might encounter a lens whose surfaces are also small sections of spherical surfaces, but which is *narrower* at the center than at the edges; this lens is a **diverging lens**. Lenses formed from sections of spherical surfaces, whether converging or diverging are called lenses **spherical lenses**. (In the initial demonstrations, you will also work with some **cylindrical lenses**, whose front and back surfaces are patches of a cylindrical surface instead of a spherical surface. While a cross sectional view of such a lens (drawn perpendicular to the front and back curved surfaces) looks just like Figure 2, a cylindrical lens will only bring to a focus those rays that happen to lie in the plane of the paper (that is, perpendicular to the curved front and back faces of the lens).

6.4 FORMING AN IMAGE WITH A CONVERGING LENS

Imagine that we place an object near the axis of a thin converging lens at a position a distance p from the lens such that $p > f$. Some of the rays scattered by a given point on the object intersect the lens, and it turns out that these rays become focused at another point to the right of the lens' focal point. Figure 3 below shows how rays from two points A and B on an arrow-shaped object are collected and focused at two corresponding points A' and B' on a plane a distance q from the center of the lens. If I could show you the trajectories of rays emanating from other points on the object, you would see that light emanating from *any* given point on the object being focused at a corresponding point in the plane on the right.

Notice that to an eye further to the right of the plane, photons from the object that go from A and B through the lens look exactly as if they had been originally emitted by points A' and B' respectively. Therefore, instead of the object appearing a distance p behind the lens, it will look to the eye as if it is floating in space upside down and a distance q in front of the lens. (You might want to *try* this during the first optics lab: it is actually a bit tricky to see). Yet the object is not really at that location, as you can verify by passing your hand through the region where the object appears to be. What we have in front of the lens is instead an **image** of the object.

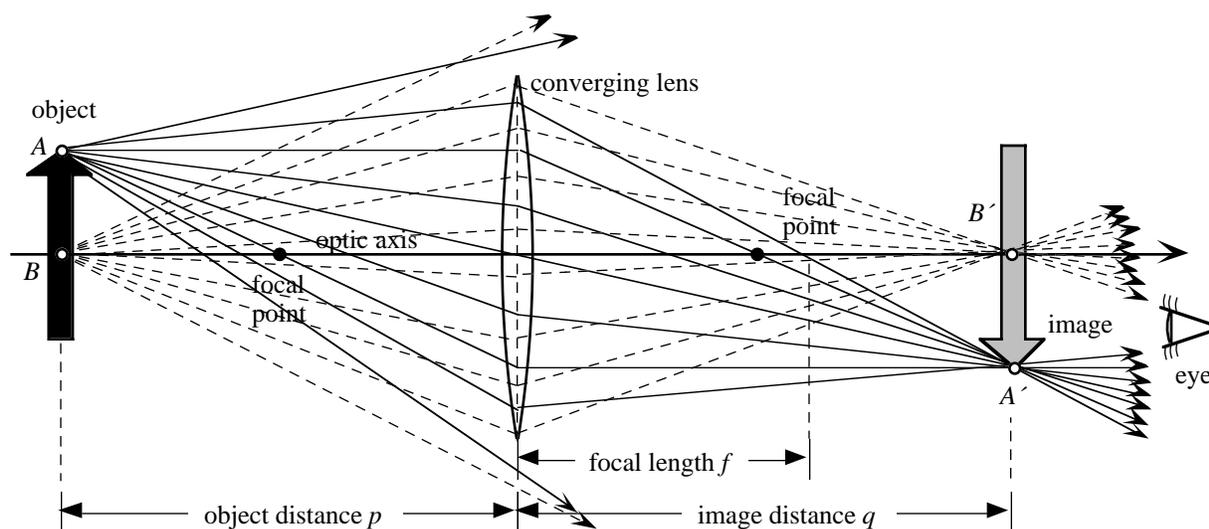


Figure 6.3: Rays that emanate from a given point on the object a distance $p > f$ from the lens are focused by the lens to a corresponding point lying on the plane perpendicular to the lens' axis and a distance $q > f$ from the lens. This creates an image of that object in that plane. Solid lines are rays from point A , and dashed lines are rays from point B . Note that not all rays coming from the object are captured and bent by the lens: most of these rays miss the lens. (The diagram shows only two such rays for each point).

[An especially vivid example of an image like this is a trick vase that you might have seen that uses curved mirrors to project an image of a coin so that it looks exactly as if the coin were floating at the mouth of the vase. When you move your hand to pick up the coin, however, you find nothing there! In the case shown in Figure 6.3, if you move your head just a little bit so that your eye no longer intercepts photons going through the lens, the image disappears. Cues like this let you know that the image that seems to be there is not in fact real. The designers of floating-coin vase have cleverly masked or eliminated most of these kinds of visual cues and masked scattered light from other sources: as a result, the floating coin image looks unusually bright and realistic.]

6.5 PRINCIPAL RAYS

Drawing a diagram like Figure 6.3 is difficult and time-consuming. We can use a simplified version of Figure 6.3, however, to quantitatively determine the location and orientation of the image formed by a converging lens.

We do this by considering three special rays emanating from a given point A on the object, rays that we can call the **principal rays** radiating from that point.

1. The *first principal ray* is the ray that goes through the center of the lens. Near the center of the lens, the front and back surfaces of the lens are essentially parallel, like an ordinary flat piece of glass. Just as photons going through a flat pane of glass are not significantly deflected, so photons going through the center of the lens are not significantly deflected: this principal ray is therefore simply a straight line.
2. The *second principal ray* is the particular ray that moves away from the point in a direction initially parallel to the axis of the lens. According to the definition of the focal point of the lens, that ray will be bent so that it goes through the focal point on the other side of the lens.
3. The *third principal ray* is that particular ray that moves away from the point in a direction initially toward the focal point on the near side of the lens: again according to the definition of the focal point, the lens will bend this ray so that it becomes *parallel* to the axis of the lens.

We are interested in these special rays because they are much easier to construct accurately than any of the other rays emanating from the point.

Figure 6.4 shows that constructing these three rays from any given point A on the object uniquely locate the corresponding point A' on the image (in a pinch, we can use any *two* of these rays to locate A'). Such a diagram is relatively simple to draw and easy to interpret. Note that if we know the focal length f and object distance p , we can use a diagram like this to determine *quantitatively* the image distance q .

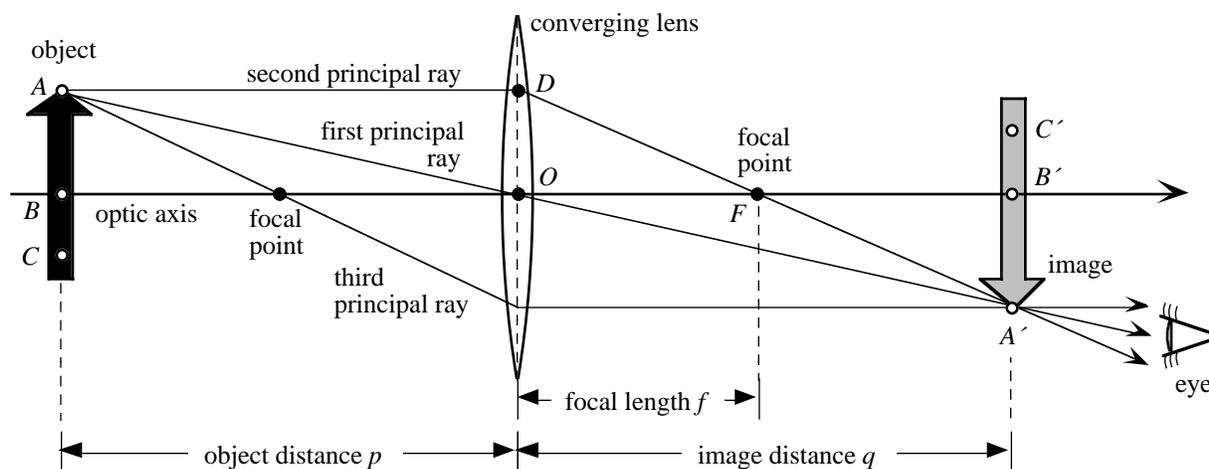


Figure 6.4: The principal rays from point A on the object can be precisely constructed if the focal length of the lens is known, and can be used to precisely locate the point A' on the image. (Note that all three principal rays for point B on the object coincide with the optic axis of the lens.)

6.6 THE THIN-LENS EQUATION

We can actually use Figure 6.4 to determine the *quantitative* relationship between the object distance p , the image distance q , and the focal length f . Note that the right triangle $DABO$ is similar to the right triangle $DA'B'O$ (since they have the same acute angle). This means that the ratio of the lengths of these triangles' short legs should be equal to the ratio of their long legs:

$$\frac{\overline{A'B'}}{\overline{AB}} = \frac{q}{p} \quad (6.1)$$

The triangles $DDOF$ and $DA'B'F$ are also similar: this means that

$$\frac{\overline{A'B'}}{\overline{DO}} = \frac{q-f}{f} = \frac{q}{f} - 1 \quad (6.2)$$

If you combine these results and note that $\overline{AB} = \overline{DO}$, you can derive a simple relationship between p , q , and f . You should complete this derivation as part of your writeup for the *Thin Lenses* lab.

EXERCISES

Exercise 6.1

On Figure 6.4, draw in the principal rays for the object point C . (The rays should converge on the image point C' .)

Exercise 6.2

Consider the drawing below. Assume that the focal length of the lens is 7 cm. Use the principal ray method to locate the position of the object's image.

