7

RAY OPTICS II

7.1 INTRODUCTION

This chapter presents a discussion of more complicated issues in ray optics that builds on and extends the ideas presented in the last chapter (which you must read first!)

7.2 VIRTUAL IMAGES

Consider a situation where we place the object closer to a converging lens than its focal length. Figure 7.1 shows a ray diagram for such a situation.

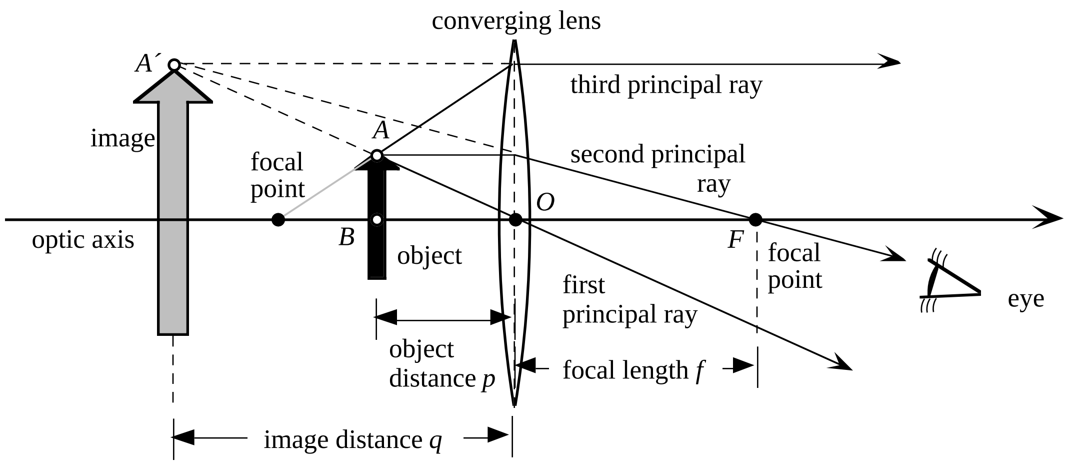


Figure 7.1: If an object is placed closer to the lens than the lens' focal length, the rays emanating from point *A* on the object never converge to the right of the lens. However, to an eye looking at these rays, it looks exactly as if the lens was not there and the rays were emanating from point *A´*. (The dashed lines in this diagram show the paths that the photons seem to follow according to the observer on the right, whose brain will automatically assume that the photons travel along straight lines.)

In this situation, the rays from point *A* on the object never actually converge anywhere to the right of the lens. On the other hand, these rays will *look* to an observer exactly as if the lens were not there and the rays were diverging from the point *A´*. Therefore, the object will look to the observer as if it were magnified in size and set further back from the lens than it actually is. (This is the principle behind the operation of a magnifying glass.)

This is a qualitatively different kind of image than the images formed by the lens in Figures 6.3 and 6.4 in the last chapter. We call an image where the rays of light radiating from a point on the object actually converge to a physical point in space (as in Figures 6.3 and 6.4) a **real image**. When the rays from an object point never actually converge toward a different physical point in space but simply appear to the eye *as if* they were radiating from an image point, we call the image a **virtual image**.

Note that to the eye, a real image and a virtual image *look* the same: in each case, the photons radiated from a specific point on the object and refracted through the lens look as if they were radiated from some *other* point in space than they actually are. Even so, these kinds of images are physically different. Here is a practical, common-sense way to distinguish between them. If the image is *real*, there ought to be a place in space where you could place an opaque white card so that the image will be projected onto that card, since light from a point on the object physically converges toward a point in a certain plane in space. On the other hand, if the image is *virtual*, there is nowhere that you could place such an opaque card to display the image.

In the *Thin Lenses* lab, you should have found both theoretically and experimentally that $(1/q) + (1/p) = (1/f)$: this is the **thin lens equation**. Even though you didn't derive it with virtual images in mind, we can still use this equation to find the virtual image's position. However, $p < f$ in this case, so $1/p > 1/f$ fi $1/q$ imust be *negative*. In this case, a negative "distance" q means that the image lies on the other side of the lens than it does in the normal situation shown in Figure 6.4.

7.3 A DIVERGING (CONCAVE) LENS

Unlike a converging (convex) lens, a **diverging** (concave) **lens** is thinner in the middle than it is at the edges. If its surfaces are small patches of a spherical surface, then a thin diverging lens has the property of diverging initially parallel rays so that they *appear* to the eye as if they were radiating from a point that we call the focal point of the diverging lens (see Figure 10.2).

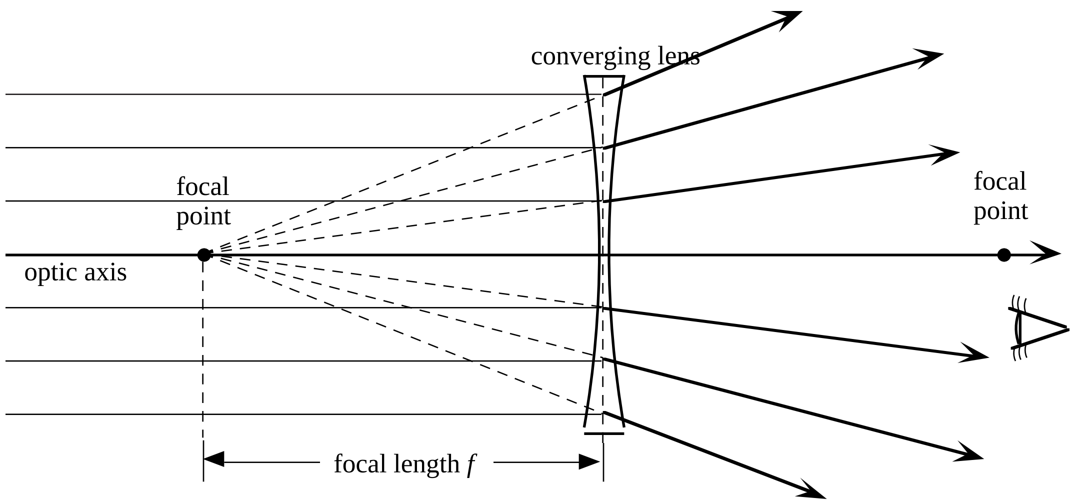


Figure 7.2: A spherical diverging lens bends initially parallel rays so that they appear to the eye to be radiating from a point that we can define to be the lens' focal point.

Just as in the case of a converging lens, we can construct a ray diagram to locate the image produced by a diverging lens, as shown in Figure 7.3.

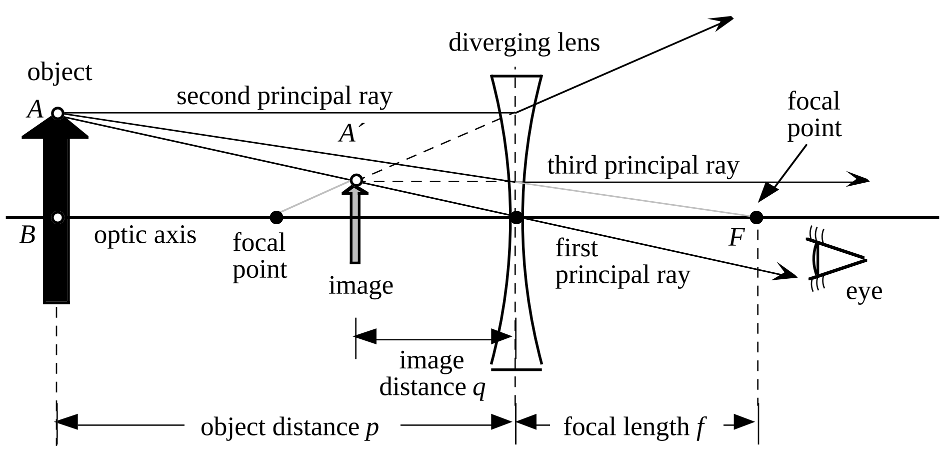


Figure 7.3: Using a ray diagram to locate the virtual image formed by a diverging lens. The dotted lines show the trajectories that the photons appear to follow, according to the observer. The gray lines indicate the relationships between the second and third principal rays and the focal points of the diverging lens.

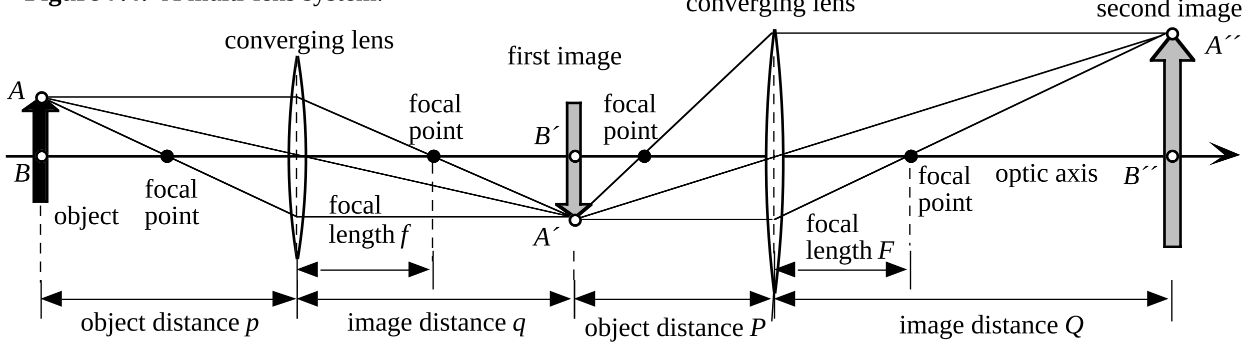
Note that a diverging lens bends the second principal ray (which initially is moving parallel to the axis of the lens), so that it appears to be radiating from the lens' focal point. The third principal ray in this case is a ray that originally is going toward the focal point on the far side of the lens: this ray is bent parallel to the axis. The image produced by a single diverging lens is *always* virtual.

The thin-lens equation describes the behavior of diverging lenses as well as converging lenses (with an appropriate choices of signs of p, q, and f). As an optional exercise (that would really test and develop your understanding), you might try to *prove* this using an argument similar to the one we used to get equations 6.1 through 6.4 in the last chapter.

7.4 MULTI-LENS SYSTEMS

We can use both the ray diagram technique and the thin-lens equation to analyze optical systems consisting of two or more lenses. The basic technique is to take the object and use the equation and/or diagram to predict the characteristics of the image produced by the *first* lens in the sequence. We then use this image as the object for the second lens in the sequence. See Figure 7.4 for an example of a two-lens ray diagram.

Figure 7.4: A multi-lens system.



Though Figure 7.4 shows a sequence of real images (mostly because the diagram is cleaner and easier to follow), we can do the same kind of analysis even if one or both images are virtual.

10.5 CONSTRUCTING A TELESCOPE (optional)

The simple magnifier shown in Figure 7.1 works well increasing the apparent angular size of nearby objects, but is not suitable for increasing the angular size of distant objects, since the object must actually be closer to the lens than the lens' focal point to be magnified in this way. To increase the apparent angular size of a distant object, we need a more complicated system.

We can construct a suitable telescope with two lenses as shown on the next page. Imagine that we have an object very far from the telescope. Rays coming from any specific point A on the object will thus be approximately parallel when they reach the first lens of the telescope (which we call the **objective** of the telescope) and will make a common small angle q_0 with respect to the axis of the telescope. The objective creates an inverted real image of the distant object exactly at its focal point. If we arrange things so that this real image is just *inside* the focal length of a second lens (the **eyepiece**), the second lens then creates an apparently larger inverted virtual image of the object for the viewer, as shown in Figure 7.5. The angular size q of this image can be much larger than the angular size of the original object q_0.

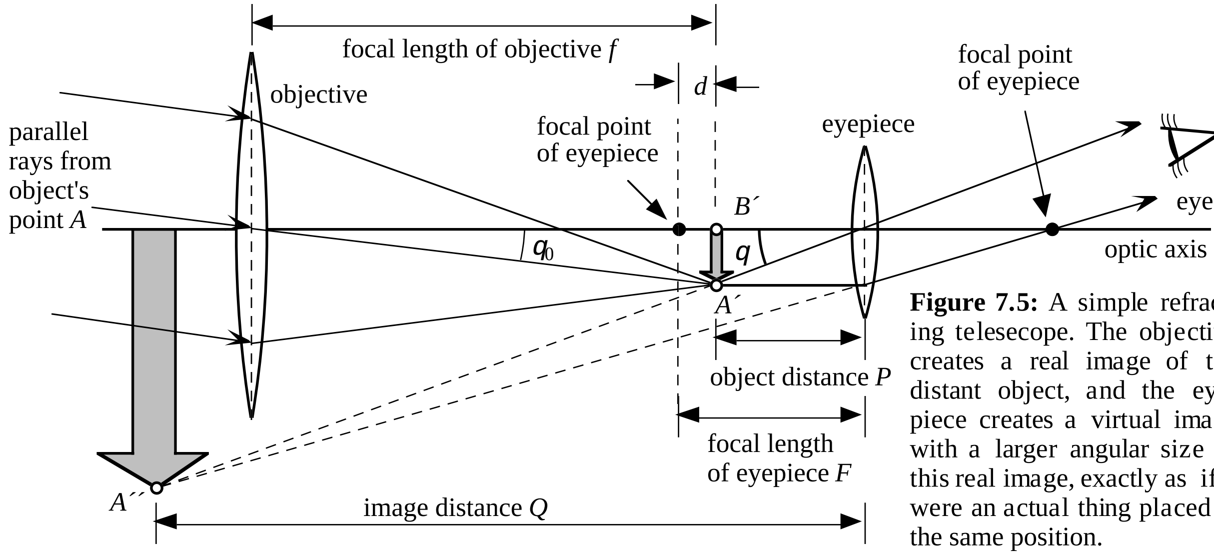


Figure 7.5: A simple refracting telesecope. The objective creates a real image of the distant object, and the eyepiece creates a virtual image with a larger angular size of this real image, exactly as if it were an actual thing placed at the same position.

The angular size of the final image will appear to the eye to be larger by a factor of θ/θ_0. If we define the distance between points A' and B' to be h', and assuming that the angles θ and θ_0 are very small so that $\tan\theta \approx \theta$ and so on (this will be true for realistic situations), then

$$\text{magnification factor } m = \frac{\theta}{\theta_0} \approx \frac{h'/(F-d)}{h'/(f+d)} = \frac{f+d}{F-d} \tag{7.1}$$

Since the real image is typically *very* close to the focal point of the eyepiece in real telescopes, d is very small compared to either f or F, and thus the magnification factor is usually $m \approx f/F$.

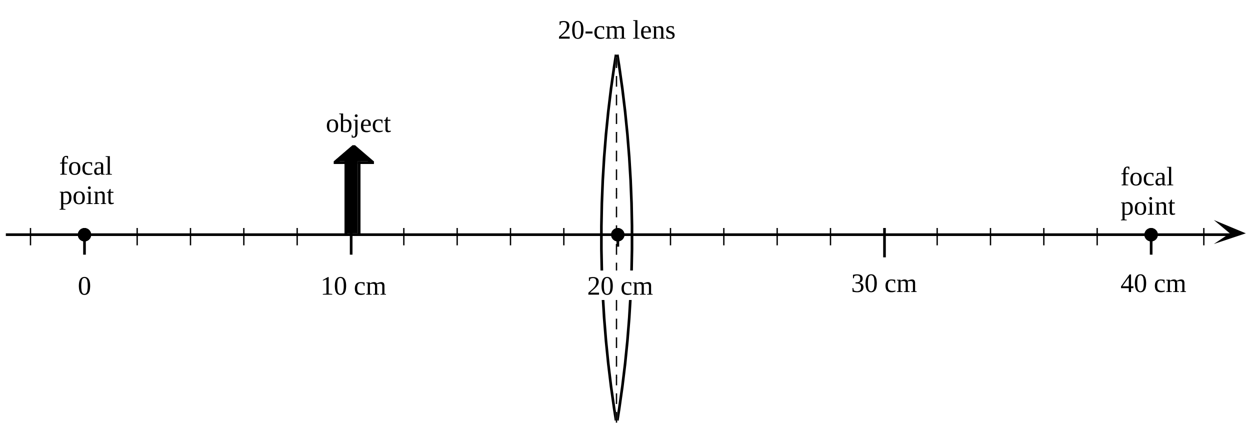
A two-lens microscope is similar in construction: the microscope objective creates a real image of the object, and the eyepiece creates an inverted virtual image of larger angular size from this real image. A two-lens microscope give larger and more controlled images than are possible with a single-lens magnifier.

EXERCISES

Exercise 7.1

Imagine that you have a convex lens with a focal point of 20 cm. While working with the optical bench, you are moved to place the object only 10 cm from the lens, as shown below.

(a) What does a ray diagram look like for this lens arrangement?



Exercise 7.1 (continued)

(b) Where does the thin-lens equation predict the image will land?

(c) Explain how we can reconcile the predictions of the ray diagram and the thin-lens equation. How would the eye interpret the rays coming from the second lens as forming an image?

(d) What kind of image is this? Can you project it on an opaque screen? Explain.

Exercise 7.2. (A system with two lenses, but no curve balls.)

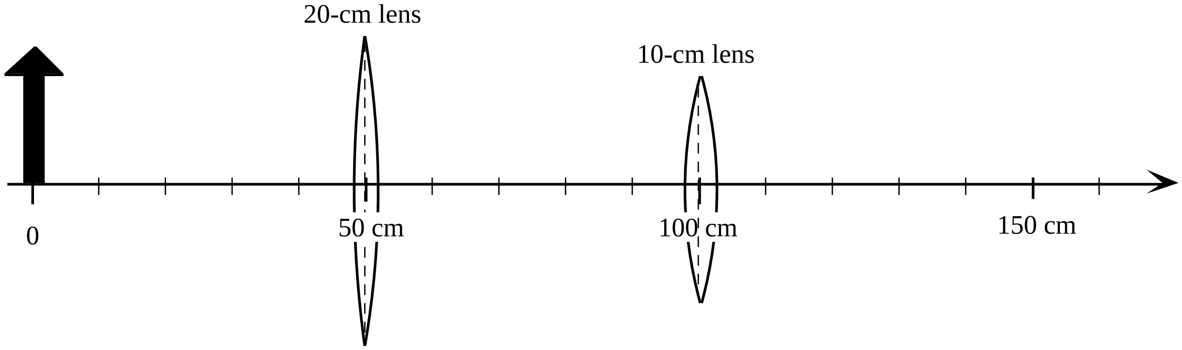
Imagine that now you are using two lenses, one whose focal length is 20 cm and one whose focal length is 10 cm. You separate the two lenses by 50 cm and then place an object 50 cm from the 20-cm lens (see the drawing below).

(a) Where does the thin-lens equation predict that the 20-cm lens will form an image?

(b) If you treat this image now as the object of the second lens, how far from that second lens is the image?

(c) Where will the image of this "object" form, according to the thin lens equation?

(d) What does the ray diagram from this setup look like? (You will have to draw in the focal points of the two lenses. Note the scale! Remember that you can get the principal rays for the second object from the location of the image formed by the first lens.)

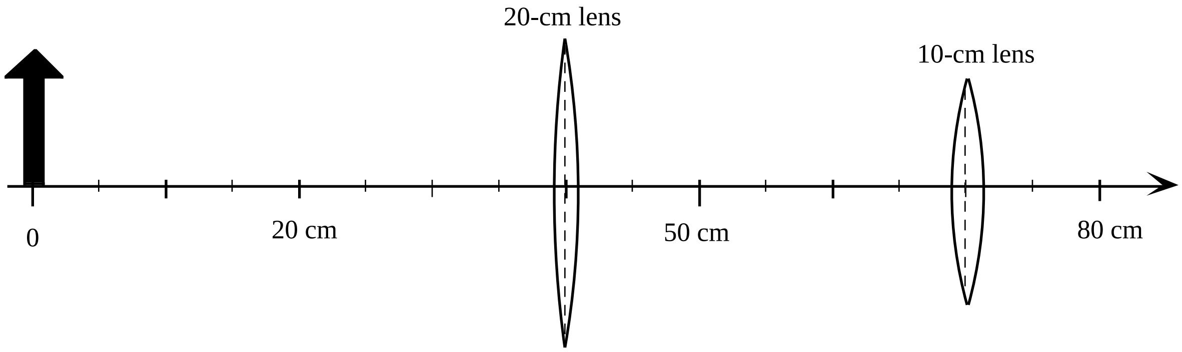


(e) Is this ray diagram consistent with your prediction using the thin-lens equation?

Exercise 7.3. (A virtual object.)

You are using your 20-cm lens and your 10-cm lens again. This time you separate the two lenses by 30 cm and you put the object 40 cm in front of the 20-cm lens. (The scale has changed again since the last drawing.)

(a) Draw the ray diagram for this situation. (Go as far as you can.)



(b) Where does the thin-lens equation predict the *first* image will form this time?

(c) How far will this image be from the second lens?

(d) What problem do you encounter trying to use this image as the object for the second lens?

(e) Your result for exercise 7.1 should suggest how the eye will interpret the diverging rays from the second lens as forming an image. Describe how to locate this second image on your diagram.

(f) Check that the location of the second image is consistent with the thin-lens equation.