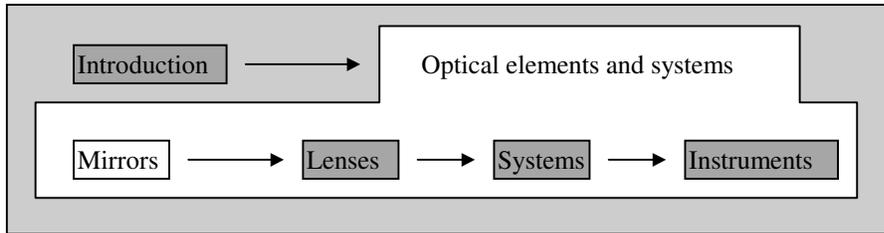


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## Reflection and Mirrors

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### O2.1 Introduction to Single Mirrors

In this section we will be introduced to a number of new ideas. We will start with mirrors because the interaction of light with mirrors, based on reflection, is easier to deal with than the interaction of light with lenses, which is based on refraction. However, we must make a special effort to understand the ideas of real and virtual images in addition to understanding the title subject of mirrors. If we finish this section with only a recipe for dealing with mirrors and not an understanding of the two types of images, we will be unprepared for future work.

**O2.2 IMAGES - VIRTUAL AND REAL.** We will introduce the ideas of real and virtual images here and revisit these ideas repeatedly. The two types of images tends to be a confusing subject, but if we use the precise definitions instead of thinking about what we “see”, the subject will be routine.

**O2.3 RAY TRACING.** We will introduce the idea of using a few critical rays to graphically understand the behavior of light and mirrors.

**O2.4 FORMULAS AND SIGN CONVENTION.** It is not sufficient to stop with the graphical solution. We will also want a quantitative answer. In order to do this consistently we will need to define certain sign conventions which we will use here and in later studies of compound systems.

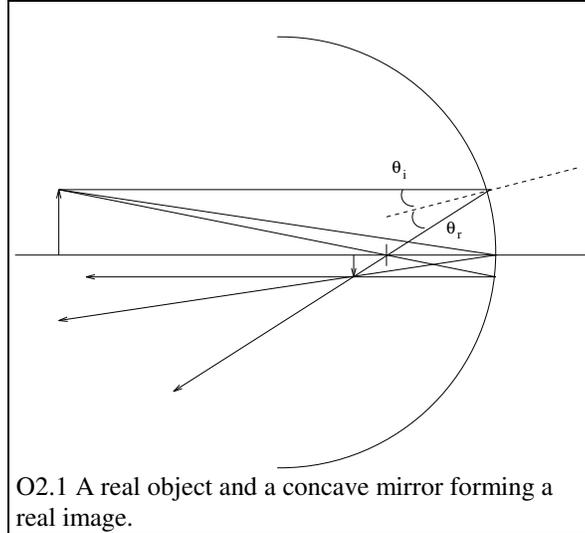
**O2.5 CONVEX MIRRORS - VIRTUAL IMAGE.** We will apply the graphical techniques and formulas to examine the behavior of light reflecting off a surface curved like the back of a spoon.

**O2.6 CONCAVE MIRRORS - VIRTUAL AND REAL IMAGES.** We will apply the graphical techniques and formulas to examine the behavior of light reflecting off a surface curved like the front of a spoon.

**O2.7 PLANE MIRRORS - VIRTUAL IMAGES.** We will finish the simple systems by examining the limiting case between convex and concave shapes.

## O2.2 Images - virtual and real.

When you look in a mirror, you see what appears to be you standing behind the mirror looking back at yourself. What makes this image appear? Let's start by looking at how an image forms, starting with a concave mirror (one in which the



reflecting surface is “caved” in). The arrow head in Figure O2.1 emits light rays in all directions. Some of the light rays strike the mirror surface and reflect back toward the arrow. The angle at which a light ray reflects is the same as the angle of incidence. As you can see, the reflected rays all pass through one point as they travel away from the mirror. If a piece of paper or screen were placed at this point, an image of the arrow head would appear. In fact, this is called a real

image.

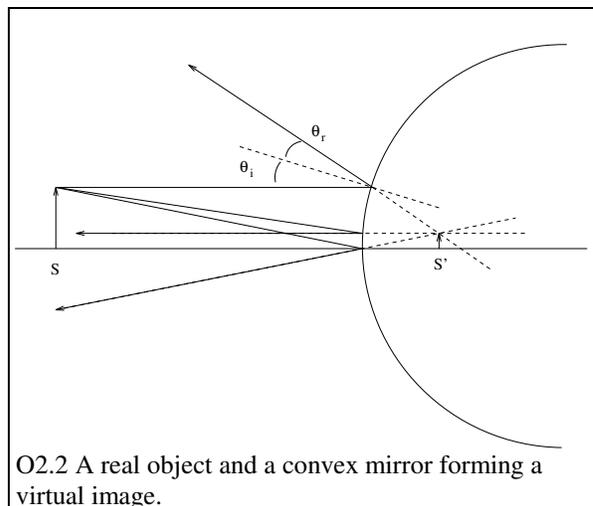
This real image appears sharp and in focus on a screen, although it may be enlarged or reduced compared to the object. Even if the screen is removed, the location where all of the light rays are converging is still called the location of the real image. The light rays really are converging at that spot.

Technically, the object in Figure O2.1 (the arrow) is called a **REAL OBJECT**. This is not because the arrow is physically located at this point, but because it is a point in space from which the light rays emanate. Light rays really leave the location of the arrow head and travel outward through space.

Why is this distinction important? In compound systems, where there are several lenses and/or mirrors, an image from one component effectively becomes the object for another. Thus, even though the arrow is not physically located at point  $S'$ , it appears as if an arrow is located there as far as the second optical component is concerned. The image from the first mirror becomes the object for the second component.

In summary, the rays converge as they approach the location of the real image. The rays diverge as they leave a real object and travel to the next optical element.

We should note that the images formed by mirrors or lenses are not always real. For example, if the arrow was placed in front of a convex mirror, as shown in Figure O2.2, the light rays, as they reflected



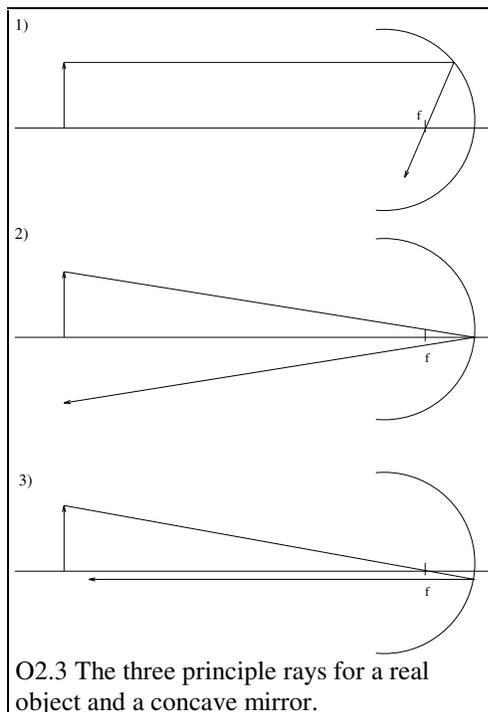
off the mirror surface, would diverge away from the mirror. There is no point at which these rays will converge. A real image cannot be formed on a screen or piece of paper. However, if we trace the reflected rays back to the right, we see that they appear to all come from one point. This point is called the location of the **VIRTUAL IMAGE**. The reflected light appears to have come from an arrow located at this point. Since light really never travels through this point, and thus could never make an image on a screen placed at this point, the image is called a virtual image. You will notice that for a virtual image, the light rays are diverging away from the image. Just as with real images, a virtual image can become the object for other optical components in compound systems.

When we deal with compound systems in O4, we will encounter situations in which there are **VIRTUAL OBJECTS**. It is difficult to describe virtual objects without concrete examples. Virtual objects occur when the rays approaching one of the latter optical elements in the system are converging as they approach the lens or mirror. We will examine the general behavior of the light rays associated with virtual objects in Section O2.3, but we will not focus on them until Chapter O4.

### O2.3 Ray tracing.

In general, we will be working with spherical mirrors because they are easily modeled mathematically. Spherical mirrors are pieces of a sphere. A radius of curvature,  $R$ , describes the surface. The radius is a very physical property of the mirror whereas the focal point, which is half of the radius, is much more useful for understanding the behavior of mirrors. If many light rays, all parallel to the optical axis, interact with a mirror they will either form a real image (if we are dealing with a concave mirror) or a virtual image (if we have a convex mirror) at the location of the focal point (The optical axis is the line through the radius and perpendicular to the surface of the mirror.) Note that for concave mirrors the focal point is located in front of the mirrored surface and is on the same side as the approaching light rays. For a convex mirror the focal point (at the location of the virtual image) is located behind the mirror. For spherical mirrors, the radius of curvature and focal point are related according to  $f = R / 2$ .

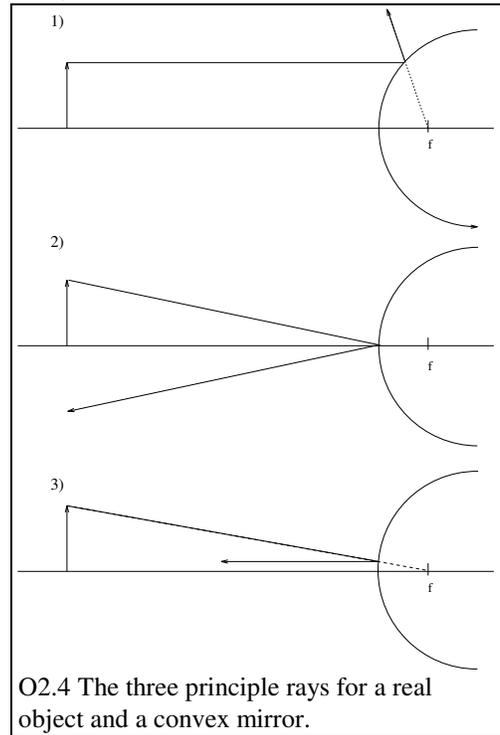
Ray tracing is a geometrical tool that helps to visualize how and where images are formed. While each point on an object emits light rays in all directions, it is most useful and easiest to follow the paths of three particular light rays in order to locate the image. The special rays are usually selected to come from the top of the object. We also assume that the object is oriented perpendicular to the optical axis so that the image will also be perpendicular to the optical axis. The three rays are 1) a ray that travels to the mirror parallel to the x-axis and then reflects off the mirror and travels through the focal point (for concave mirrors) or as if it had come from the focal point (convex mirrors), 2) a ray that



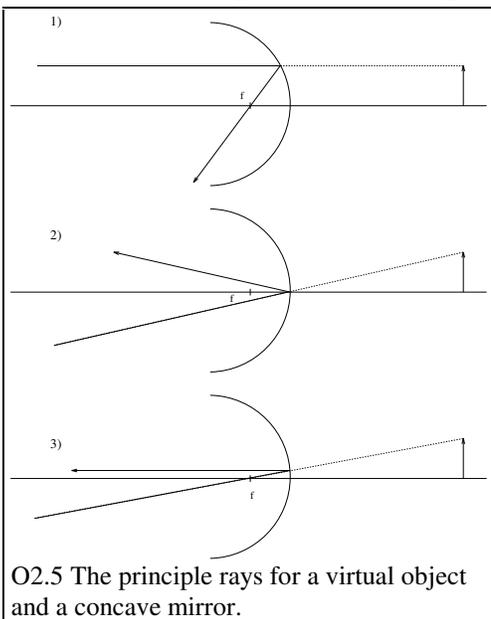
travels from the object to the point where the x-axis intersects the mirror and reflects off at an equal angle, and 3) a ray aimed through the focal point (concave mirrors) or aimed at the focal point (convex mirrors) which reflects off the mirror and heads off parallel to the optical axis.

The previous discussion defined several terms. Now let us look at how things actually work. The three rays are illustrated separately for concave and convex mirrors and for real and virtual objects in Figures. O2.3 to O2.6. Usually, we will draw the three useful rays on a single diagram but for clarity they are shown individually in these figures. Figure O2.3 shows a real object and a concave mirror. The ray shown in the top part of the figure enters parallel to the optical axis and exits through the focal point. In the middle part of the figure we see a ray that reflects off the center of the mirror with equal angles of incidence and reflection. (When drawing this ray, pretend the object is inverted and draw the outgoing ray through the pretend arrow head.) The ray in the bottom part enters through the focal point and exits parallel to the optical axis. Actually, all of these rays have equal angles of incidence and reflection, but it is most obvious for this last case.

Figure O2.4 shows a real object and a convex mirror. The same three rays are shown, but we also notice some dashed lines. Since the focal point is located behind the mirror, the ray in the top part of the figure cannot actually pass through the focal point. However, it is still related to the focal point and travels as if it had come from the focal point. In the third part, we see that the correct way to aim the



O2.4 The three principle rays for a real object and a convex mirror.



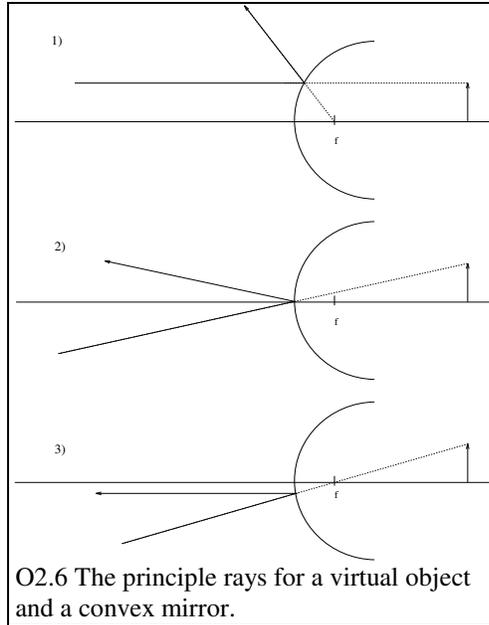
O2.5 The principle rays for a virtual object and a concave mirror.

incoming ray is to draw it as if it would go through the focal point if it were not intercepted by the mirror.

In Figure O2.5 we have a concave mirror with a virtual object. This will never be the first step of a mirror problem and can only occur when dealing with problems containing multiple optical elements. We will not see this situation again until O4, when it will be crucial. Notice that the incoming rays are converging and would all intersect at the arrow if the mirror did not get in the way. However, the behavior of the three useful rays is unchanged; a ray that goes in parallel goes out through the focal point, a ray that goes in at the

center goes out with equal angles, and a ray that goes in through the focal point goes out parallel to the axis.

The last case, a convex mirror and virtual object, is illustrated in Figure O2.6. Notice that because the object is virtual, the incoming rays are trying to converge at the tip of the arrow. Note too, that because the focal point is behind the mirror, the outgoing ray in the top part of the figure acts as if it came from the focal point and the incoming ray in the bottom is directed at the focal point but never actually passes through it.



## O2.4 Formulas and sign convention.

This section will lay out the mathematical means of finding and characterizing images. The methods of this section and the previous section provide us with the means of finding images in a reliable fashion.

The behavior of the optical system is easily expressed in a simple formula;

$$\frac{1}{S} + \frac{1}{S'} = \frac{2}{R} = \frac{1}{f}$$

where  $S$  is the distance from the object to the mirror,  $S'$  is the distance from the mirror to the image,  $R$  is the radius of curvature of the mirror, and  $f$  is the focal length. So far this seems straightforward, but we need to keep the sign of each of these quantities straight. The table below and the discussion that follows should help us to keep things straight.

Real Object (diverging light rays)	$S > 0$ (object in front of the mirror)
Real Image (converging light rays)	$S' > 0$ (image in front of the mirror)
Virtual Image (diverging light rays)	$S' < 0$ (image in back of the mirror)
Virtual Object (converging light rays)	$S < 0$ (object in back of the mirror)

We will state the various conditions in several equivalent ways involving different words. This is not intended to confuse, but rather to reinforce the ideas behind the sign conventions. We will agree to treat  $S$ , the distance from the object to the mirror, as a positive value if we have a real object located in front of the mirror. In this case the light rays are diverging as they leave the real object and

approach the mirror. Unlike mechanics, it really does not matter whether the rays are moving to the right or left. What matters is the location of the object relative to the mirror or the divergence of the rays. Virtual objects will have a negative value for  $S$ .

For  $S'$ , we will take the value to be positive if the rays leaving the mirror are converging. These converging rays will intersect somewhere in front of the mirror. The focused image will be on the same side as the real object, in front of the mirror, and since the rays are converging, the resulting image is real. If we find that  $S'$  is a negative number, then the image is located on the side of the mirror opposite the object, the rays will be diverging, and the image will be virtual.

The sign convention for  $R$  and  $f$  are the easiest.  $R$  and  $f$  are positive for concave mirrors and negative for convex mirrors. If these are positive, then the focal point is on the same side of the mirror as the incident light rays and the real object. If they are negative (convex mirrors), the focal point is located behind the mirror.

The image, either real or virtual, is usually a different size than the original object. The ratio of the size of the image to that of the object is called the magnification. In addition, the image may be inverted, in which case the magnification will be negative. The magnification is related to the object distance  $S$  and image distance  $S'$  by the expression

$$M = -S'/S$$

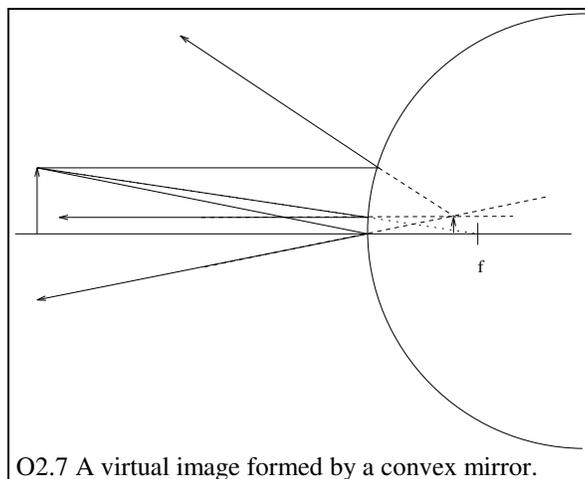
where we can get the correct sign of  $M$  if we use the previous sign conventions for  $S$  and  $S'$ . In terms of the height of the object and image,

$$M = +h'/h$$

where  $h$  and  $h'$  are the heights of the object and image, respectively.

## O2.5 Convex mirrors - virtual image.

We now apply the ray diagrams and sign conventions to a single convex mirror. Figure O2.7 shows an object and convex mirror along with the three principle rays. For purposes of discussion, let us assume that the radius of curvature is 20 cm (a very standard convex mirror), and the object is 30 cm from the mirror. The ray diagram indicates that the image is virtual, erect, and located -7.5 cm from the mirror. It is important to see that the image is virtual by the definition given above. Since the light rays do not actually converge to a single point after reflecting off the mirror, the image cannot be real. However, note that after reflecting off the mirror, the rays move AS IF they had been emitted from a fictitious object located



O2.7 A virtual image formed by a convex mirror.

behind the mirror. Would we **see** anything if we looked at this mirror? Yes, but as we said above, seeing has little to do with the ideas of virtual or real images. Is there any place to put a screen that would show an image? Certainly, in front of the mirror there is no place where the rays cross and so there is no place to put a screen. We might be tempted to say that we could place our

screen behind the mirror but this is silly because the light never really goes through the mirror. We can draw dashed lines back there, but they do not really exist.

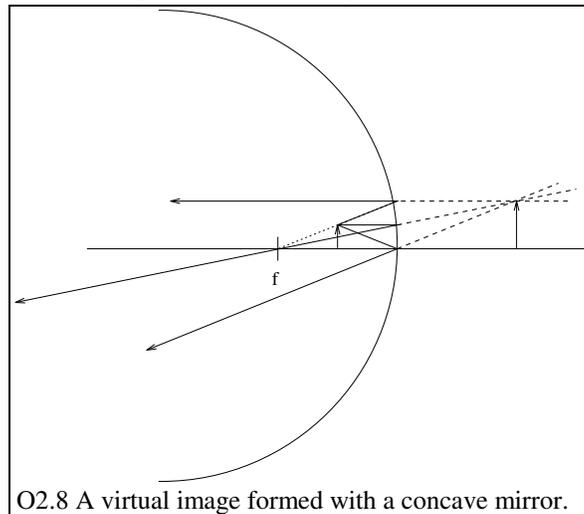
The system can also be evaluated with the approach described in section O2.4. The object distance,  $S$ , is +30 cm because the rays are diverging as they leave the object. The focal distance is -10 cm where the minus sign indicates that the mirror is convex. The relation from O2.4 yields -7.5 cm for the image distance,  $S'$ , and the magnification is +0.25. The minus sign for  $S'$  indicates that the image is virtual and located behind the mirror. The image is smaller than the object but is erect or right-side-up.

Thus, the two methods are in agreement.

## O2.6 Concave mirrors - virtual and real images.

Concave mirrors are somewhat more involved. Concave mirrors can yield either real or virtual images depending on the position of the object relative to location of the focal point. We will see that if the object is positioned outside the focal point a real, inverted image results. We will also find that if the object is placed close to the mirror, inside the focal point, a virtual image is the result.

We will start with an example of a concave mirror of radius 20 cm with an object placed to form a virtual image. Because the mirror is concave, the focal length,  $f$ , is +10 cm. Let us put an object 5 cm from the mirror. Figure O2.8 shows the three principle rays from the top of the object. From the figure, it appears that a virtual image is formed. Again, how do we know that this image is virtual? It is virtual because the light rays



O2.8 A virtual image formed with a concave mirror.

diverge and do not physically cross after reflecting off the mirror. The rays only seem to come from a point behind the mirror. We can also see that the image is larger than the original object.

The numerical approach to this problem yields the same conclusions. With the values here we have

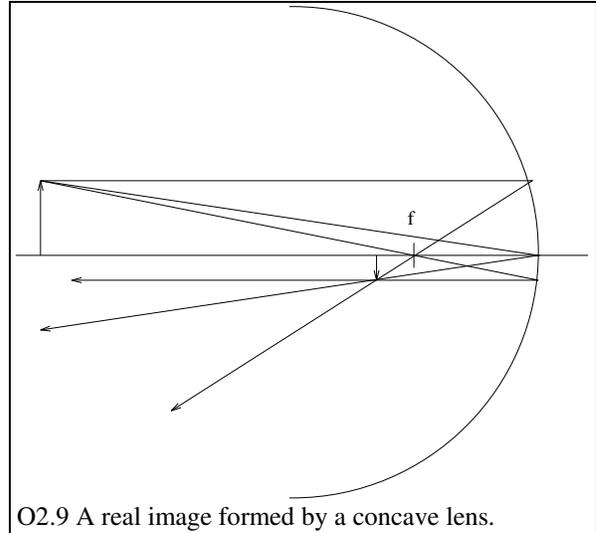
$$\frac{1}{5 \text{ cm}} + \frac{1}{S'} = \frac{1}{10 \text{ cm}} \Rightarrow S' = -10 \text{ cm} \text{ and}$$

$$M = -\left(\frac{-10 \text{ cm}}{5 \text{ cm}}\right) = 2.$$

We see that the virtual image ( $S'$  negative) is located 10 cm from the lens and positioned behind the lens. The image is oriented the same way as the object, erect in this case, because  $M$  is positive and the image is twice the size of the object.

The next example will use the same mirror but place the object 40 cm from the mirror. Again we start by constructing a ray diagram that is shown in Figure O2.9. For the first time in any of our examples, we see that the rays do in fact all cross at a point below the axis about 14 cm in front of the mirror. This is a **real** image! Yes, if we looked with our eye we would see a small reflection of the object. If we put a screen at this location (being careful to let some rays still travel from the

object to the mirror), a little image would be observed on the screen just as if we were using an overhead projector. Looking at the ray diagram, we expect that when we find a numerical answer, we will see that  $S'$  is about +14 cm, the magnification is negative but less than one.



Let us find the numerical solution as before:

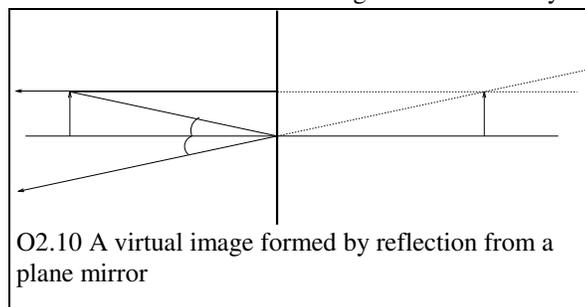
$$\frac{1}{40 \text{ cm}} + \frac{1}{S'} = \frac{1}{10 \text{ cm}} \Rightarrow S' = 13.33 \text{ cm} \text{ and}$$

$$M = -\left(\frac{13.33 \text{ cm}}{40 \text{ cm}}\right) = -0.33.$$

Just as we expected, the image is located at a positive distance so that it is in front of the mirror, real, and at a distance consistent with the ray diagram. The magnification is negative, implying that the object and image are oriented differently, and the magnitude of the magnification is less than one telling us the image is smaller than the object.

## O2.7 Plane mirrors.

Plane mirrors could easily be grouped with convex mirrors as there is no way to form a real image with a plane mirror. There is a focal point but since the radius of curvature for a plane mirror is infinite, the focus is located infinitely far away from the mirror. This means that we need **really big** pieces of paper to draw the ray diagrams. Another solution is to only draw two of the three rays: the ray that comes in parallel to the axis and goes out exactly back along the path parallel to the axis and the ray that reflects off the center of the mirror. (The ray that comes in parallel to the axis really goes out through the focal point but since  $f$  is behind the mirror at negative infinity, the ray reflects back along the original path.) The result is shown in Figure O2.10 but the reality is that for a plane mirror, one always has a virtual image that is located behind the mirror at the same distance that the object is located from the mirror. The magnification is always exactly one.



Numerically, one can simply take  $M=1$  and  $S'=-S$ , although the early formula is still valid if we realize that  $1/f=0$  if  $f$  is infinite.

$$\frac{1}{S} + \frac{1}{S'} = 0.$$

**O2.8 HOMEWORK PROBLEMS**

1. A convex mirror has a radius of 20 cm. An object is placed 50 cm from the mirror. Find the location of the image, the magnification, and the type of image formed.
2. For the previous mirror system, draw the three principle rays and compare your results to that obtained for the previous problem.
3. A convex mirror has a radius of 60 cm. An object is placed 200 cm from the mirror. Find the location of the image, the magnification, and the type of image formed.
4. For the previous mirror system, draw the three principle rays and compare your results to that obtained for the previous problem.
5. A concave mirror has a radius of 40 cm. An object is placed infinitely far from the mirror. Find the location of the image, the magnification, and the type of image formed.
6. For the previous mirror system, draw the three principle rays and compare your results to that obtained for the previous problem.
7. A convex mirror has a radius of 40 cm. An object is placed 100 cm from the mirror. Find the location of the image, the magnification, and the type of image formed.
8. For the previous mirror system, draw the three principle rays and compare your results to that obtained for the previous problem.
9. A convex mirror has a radius of 40 cm. An object is placed 20 cm from the mirror. Find the location of the image, the magnification, and the type of image formed.
10. For the previous mirror system, draw the three principle rays and compare your results to that obtained for the previous problem.
11. A convex mirror has a radius of 40 cm. An object is placed 10 cm from the mirror. Find the location of the image, the magnification, and the type of image formed.
12. For the previous mirror system, draw the three principle rays and compare your results to that obtained for the previous problem.
13. Assume that you are standing 1 m from a plane mirror. How far away does the image of your face appear?

