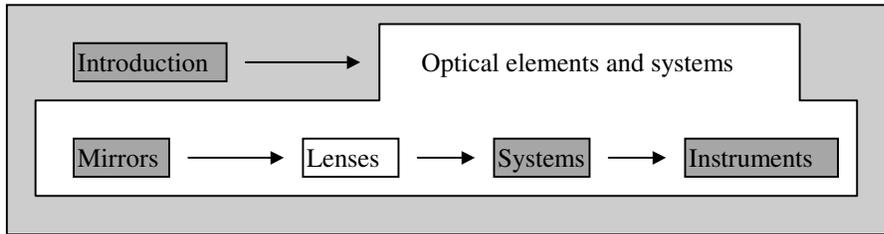

Introduction to Lenses



O3.1 Introduction to Lenses

In this section we will learn to understand and predict the behavior of single thin lenses and simple systems of such lenses. We will see that lenses can form virtual or real images. We will learn how to draw ray diagrams to obtain a visual feeling for the behavior of the lens. Finally, we will develop numerical techniques to complement the ray tracing techniques.

O3.2 RAY TRACING AND NUMERICAL APPROACHES. The ray tracing techniques for lenses are very similar to those we used for mirrors. The numerical techniques for lenses are also similar to those for mirrors. However, care needs to be taken as there are a few critical differences.

O3.3 DIVERGING LENSES - VIRTUAL IMAGES. We will apply graphical and numerical techniques to optical systems composed of a single diverging lens.

O3.3 CONVERGING LENSES - VIRTUAL AND REAL IMAGES. We will finish our study of simple lenses by an examination of the images formed by thin converging lenses.

O3.4 THICK LENSES. Almost all cases that we will encounter in life will deal with lenses that are thin. That is, they are thin compared to the other distances in the problem. If the lenses are thin then we usually ignore the fact that the refraction at the front and back surfaces of the lens do not really take place at the same location. However, there are times when the lens is thick or the required precision of the system is such that this approximation is not valid. In this section, we will briefly examine thick lenses.

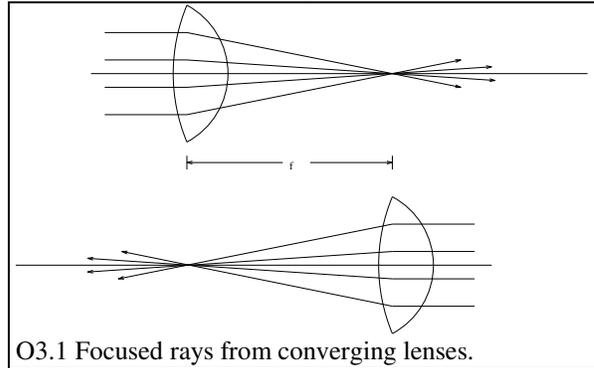
O3.2 Ray Tracing and Numerical Approaches.

Lenses, like mirrors, bend light. Unlike mirrors, the bending is done through refraction, not reflection. Another difference between lenses and mirrors is that lenses have no front or back. Light rays are bent in the same way regardless of which side they “hit” first. For this reason, each lens has two focal points, one on each side of the lens.

Converging lenses are thicker in the center than at the edges. This causes light rays which come in parallel to the optical axis from the left to focus to a point (real image) on the right side of the lens (Figure O3.1). If parallel rays come in from

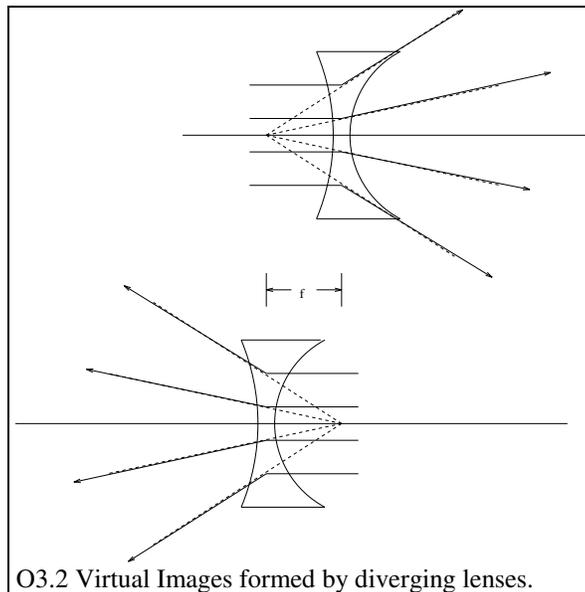
the right, they will focus to a point on the left (Figure O3.1). The focal point on the right is exactly as far from the center of the lens as the focal point on the left. Note that even if the lens shape is not symmetric, the two focal points are.

Diverging lenses are thinner in the center than at the edges. This causes light rays, which come in parallel to the optical axis from the left, to diverge as if they came from a point on the left side of the lens (Figure O3.2). If parallel rays come in from the right, they will diverge as if they



O3.1 Focused rays from converging lenses.

came from a point on the right (Figure O3.2). The focal point on the right is exactly as far from the center of the lens as the focal point on the left. Note that even if the shape is not symmetric, the two focal point locations are.



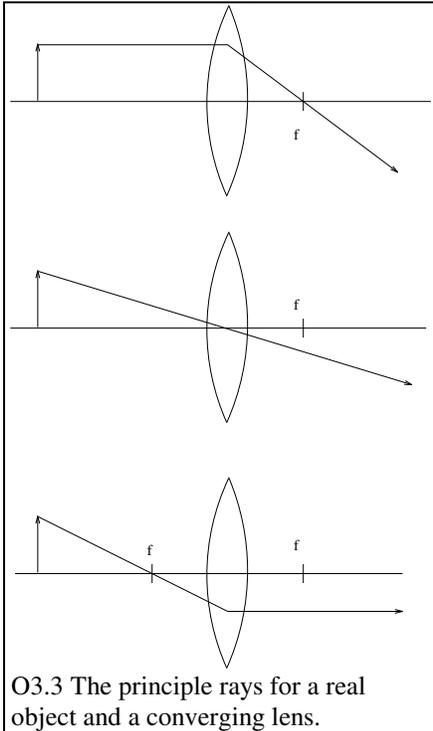
O3.2 Virtual Images formed by diverging lenses.

Just as for mirrors, ray tracing for lenses deals with three of the infinite number of rays being emitted by an object. One of these rays will travel parallel to the optical axis before interacting with the lens. One of the rays will travel along a path associated with the focal points. Finally, one of the rays will pass through the center of the lens. Figures O3.3 to O3.6 illustrate the useful rays for four optical situations: a converging lens with a real object, a converging lens with a virtual object, a diverging lens with a real object, and a diverging lens with a virtual object. We note that, just as for mirrors, the rays incident on a lens from a virtual object (Figures O3.4 and O3.6) are converging as they approach the lens and would, if the lens was not in the way, eventually focus at the location of the virtual object. We also notice that depending on the situation, the ray which comes in parallel actually passes through a focal point on the way out (Figures O3.2 and O3.3). Other times, (Figures O3.5 and O3.6) the incoming parallel ray goes out as if it came from a focal point. Similarly, sometimes the incoming ray actually passes through a focal point and sometimes the incoming ray is aimed at the focal point on the far side of the lens. When in doubt, remember that diverging lenses bend the light away from the optical axis while converging lenses bend the light toward the optical axis.

When in doubt, remember that diverging lenses bend the light away from the optical axis while converging lenses bend the light toward the optical axis.

The first special ray for ray tracing is the easiest. A ray directed through the center of the lens, where the lens intersects the axis, proceeds undeflected. This is illustrated in the middle part of Figures O3.3 to O3.6.

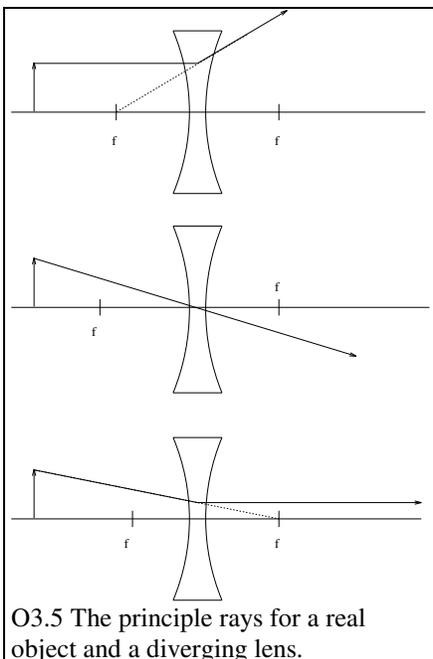
The next special ray is a bit more involved. A ray that goes toward the lens parallel to the optical axis proceeds away from the lens **related** to the focal point. If the lens is a converging lens, then the ray will bend toward the focal point on the far



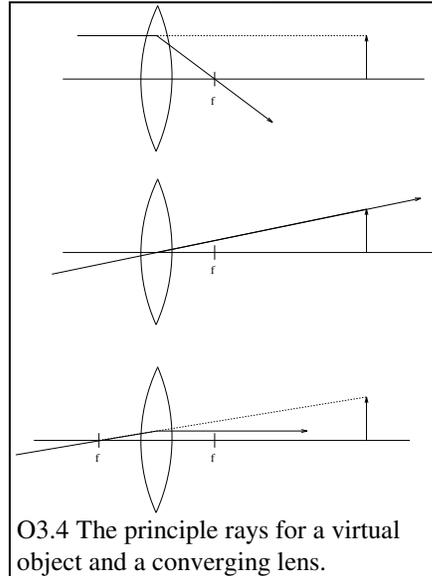
O3.3 The principle rays for a real object and a converging lens.

side of the lens. This ray will be bent toward the optical axis. If the lens is a diverging lens, the ray will be bent away from the axis after passing through the lens. This ray will be directed as if it came from the focal point on the side of the lens facing the object. A diverging lens will always bend the light ray away from the axis while a converging lens (thicker in the middle) will bend the rays toward the axis. These cases are shown in the top parts of Figures O3.2 to O3.6.

The third special ray requires the most thought. This ray will proceed parallel to the axis after the ray passes through the lens. However, the path to the lens will depend on the type of lens and the location of the object. If the lens is a



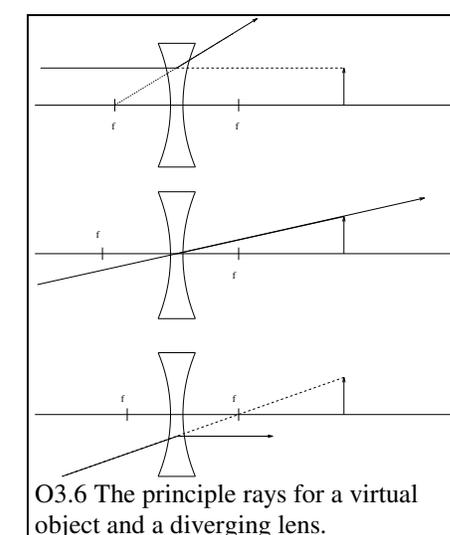
O3.5 The principle rays for a real object and a diverging lens.



O3.4 The principle rays for a virtual object and a converging lens.

diverging lens. This ray will be bent toward the optical axis. If the lens is a diverging lens, the ray will be bent away from the axis after passing through the lens. This ray will be directed as if it

came from the focal point on the side of the lens facing the object. A diverging lens will always bend the light ray away from the axis while a converging lens (thicker in the middle) will bend the rays toward the axis. These cases are shown in the top parts of Figures O3.2 to O3.6.



O3.6 The principle rays for a virtual object and a diverging lens.

intersects the lens and is redirected parallel to the axis before reaching this point. Note that as we wanted, the ray did bend away from the axis. These conditions are illustrated in the bottom portions of Figure O3.5 and O3.6. If the system under study is a converging lens, we should direct the ray from the object to the lens while going through the focal point on the near side of the lens if possible (as in Figures O3.3 and O3.4). If going from the object through the focal point on the same side of the lens would require the ray to head away from the lens (as in the second example with Figure O3.8), then aim the ray as if it came from the focal point, passed through the tip of the arrow, interacted with the lens, and proceeded parallel to the axis. Note that for a converging lens this ray is bent toward the axis in each case.

The rays never ever bounce backward or reflect a ray off a lens. If we find ourselves drawing this type of ray, it means that we are confused and need to revisit the behavior of the useful rays.

These rules need to be used with care in those instances where we encounter a virtual object. Rather than complicate the situation here, we will see how to handle this situation in some later examples.

The numerical approach is very similar to that of mirrors. The object distance, image distance, and focal length are related by

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} .$$

Real Object (diverging light rays)	$S > 0$ (object on side where light is produced)
Real Image (converging light rays)	$S' > 0$ (image on side where light is going)
Virtual Image (diverging light rays)	$S' < 0$ (image on side where light is produced)
Virtual Object (converging light rays)	$S < 0$ (object on side where light is going)

The signs of these quantities are critical. The focal length is positive for converging lenses and negative for diverging lenses. The object distance, S , is positive for real objects with emitted light rays that diverge as they leave the object and approach the lens. The object distance is taken to be negative in those isolated cases where the rays are converging as they approach the lens. Finally, if S' is positive, the image is formed on the far side of the lens and will be a real image. If S' is negative, the image is a virtual image and will be located on the same side of the lens as the object, the near side. As with mirrors, the magnification is given by

$$M = -\frac{S'}{S} = \frac{h'}{h} .$$

O3.3 Diverging Lenses - Virtual Images

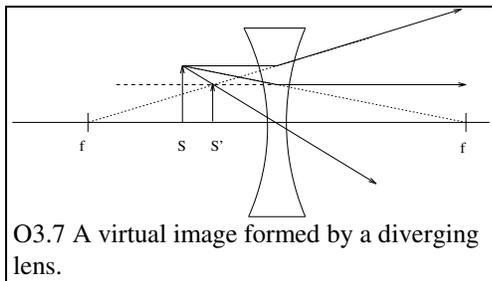
Figure O3.7 illustrates a ray diagram for a diverging lens of focal length 50 cm and an object distance of 25 cm. The three rays show that a virtual image is formed which is right side up, shorter than the object, and located about 17 cm from the lens.

The numerical solution,

$$\frac{1}{50 \text{ cm}} + \frac{1}{S'} = \frac{1}{-25 \text{ cm}} \Rightarrow S' = -16.7 \text{ cm}$$

agrees with the diagram. Note that the focal length was taken to be negative because the lens is a diverging lens. We also see that the image distance is negative as expected from the ray diagram that implies a virtual image. The numerical calculation of the magnification,

$$M = -\left(\frac{-16.7 \text{ cm}}{50 \text{ cm}}\right) = 0.333$$

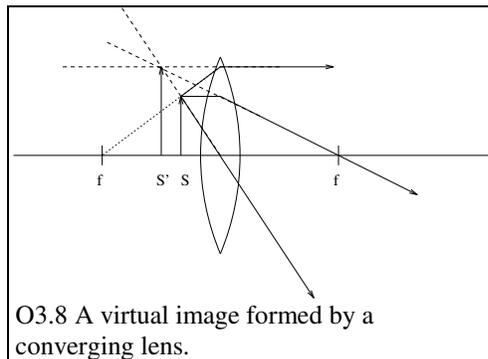


also agrees with the ray diagram since a positive value indicates that the image is oriented the same as the object and not upside down. The fact that the magnitude of the magnification is less than one tells us that the image is smaller than the object which also agrees with the ray diagram.

O3.4 Converging Lenses - Virtual and Real Images

Converging lenses can result in either virtual or real images just as concave mirrors can produce either type of image depending on the relative location of the focal point and the object. We begin by looking at the case where the object is positioned inside the focal point of the lens.

Figure O3.8 shows the three principle rays for a converging lens with a focal length of 30 cm and an object located 10 cm from the center of the lens. We see that the rays diverge after passing through the lens, but that they **appear** to come from a single point on the object side of the lens. Thus the image formed is virtual, located 15 cm from the lens, located on the object side of the lens, oriented the same way as the object, and larger.

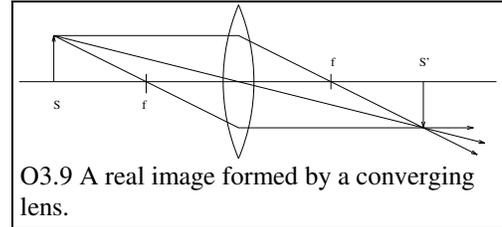


We again turn to the numerical approach to verify these results. Since this lens is a converging lens, we will take the focal length to be positive according to the sign conventions. We find that

$$\frac{1}{10 \text{ cm}} + \frac{1}{S'} = \frac{1}{30 \text{ cm}} \Rightarrow S' = -15 \text{ cm} \text{ and } M = -\left(\frac{-15 \text{ cm}}{10 \text{ cm}}\right) = 1.5$$

which agrees with our ray diagram. According to our sign conventions, the negative image distance indicates a virtual image on the object side of the lens and the magnification value indicates an image which is erect (oriented the same as the object) and larger. Because the ray diagram and numerical approach are in agreement, we have confidence that we solved the problem correctly.

Figure O3.9 shows the three principle rays for the same converging lens ($f = 30\text{ cm}$), but this time we will place the object outside the focal point at a distance of 60 cm from the lens. In this case, we observe that the three rays really do cross or converge to a single point 60 cm on the far side of the lens. We also note that the image formed is oriented oppositely (inverted) from the object. This is the first **real** image we have seen resulting from a thin lens.



As always, we verify our understanding of the system with a numerical calculation. The numerical solution

$$\frac{1}{60\text{ cm}} + \frac{1}{S'} = \frac{1}{30\text{ cm}} \Rightarrow S' = 60\text{ cm}$$

is made following the sign convention and taking the focal length to be positive. The fact that the image distance is positive indicates that the image is real and the rays are actually converging after passing through the lens. The magnification is given by

$$M = -S'/S = -\left(\frac{60\text{ cm}}{60\text{ cm}}\right) = -1$$

and we see that since this is negative indicating that the image is inverted from the original object. We also see that the object is positioned in such a way that the image is the same size as the object. Because the diagram and numerical calculation agree, we believe the solution to the problem.

O3.5 Thick lenses

Up to this point, we have dealt with ideal lenses that are thin compared the other distances in the system. Basically, we have been treating the lenses as if they have no thickness but are still able to refract the rays. Thick lenses are handled by two successive applications of techniques which find images from the refraction of light at curved surfaces. Of course, the nice relations that we have been using are ultimately based on the ideas which will be developed here. The radius of curvature of each side of the lens and the index of refraction of the material really determine the properties of the lens and the location of the focal point.

When we work with images formed by refraction, we cannot ignore the specific index of refraction of each material and the shape of the interface. We also find that while it is possible to construct ray diagrams for images formed by refraction, it is very tedious and requires very careful numerical measurements of angles. We will draw diagrams to illustrate the paths taken by some of the relevant rays, but in general, we will rely on numerical calculations and sign conventions. Snell's law can be used to derive (although we will not do so here) a relation between S and S' based on the indices of refraction and the radius of curvature of the interface. The relation is found to be

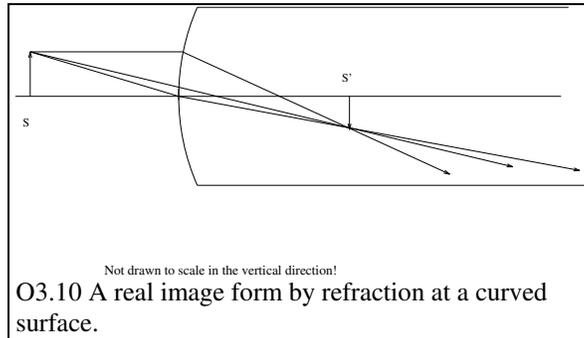
$$\frac{n_1}{S} + \frac{n_2}{S'} = \frac{n_2 - n_1}{R}$$

where n_1 is the index of refraction for the medium where the rays originate and n_2 is the index on the far side of the interface between the two materials. The sign conventions for S and S' are the same as for thin lenses. The radius R is taken to be positive if the rays see a convex surface and negative if the rays encounter a concave surface. If the surface is planar, the radius is infinite and the right-hand term in the equation above becomes zero.

Three examples will be helpful to illustrate the formation of images by refraction. The first example is shown in Figure O3.10 where we examine the image formed as light

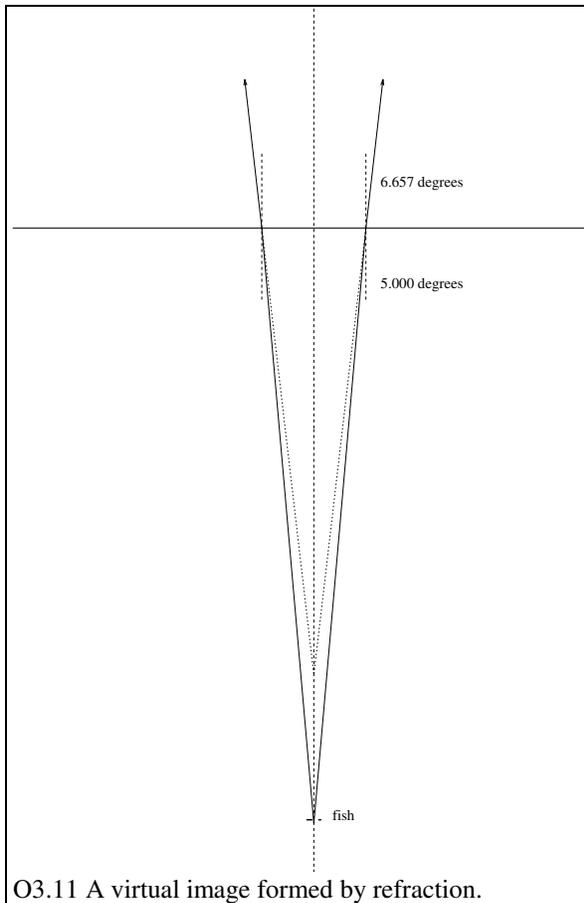
rays pass from air into glass. The interface is spherical and convex. The index of refraction of air is n_1 and has the value of 1.0 and we will take the index of the glass to be 1.6 for n_2 . The radius of curvature for the interface is 50 cm. This configuration will yield an image according to

$$\frac{1.0}{200 \text{ cm}} + \frac{1.6}{S'} = \frac{1.6 - 1.0}{50 \text{ cm}} \Rightarrow S' = 228.57 \text{ cm}.$$



O3.10 A real image formed by refraction at a curved surface.

The interface is spherical and convex. The index of refraction of air is n_1 and has the value of 1.0 and we will take the index of the glass to be 1.6 for n_2 . The radius of curvature for the interface is 50 cm. This configuration will yield an image according to



O3.11 A virtual image formed by refraction.

The sign conventions were applied in choosing the radius to be positive and in interpreting the positive result for S' as implying a real image formed inside the glass. We will agree that n_1 is the index for the medium where the rays begin and n_2 is the index for the medium into which the rays travel.

For our second example, illustrated in Figure O3.11, we will revisit the situation of light rays originating from a fish in the bottom of a pool. In this case n_1 is 1.33 because the light rays begin in the water while n_2 is 1.0 because the light ends in air. The radius of curvature for the flat surface of the water-air interface is infinite so that the right hand side of the

expression at the top of the page will be zero. If the fish, which is our object, is 200 cm below the surface then we find that

$$\frac{1.33}{200 \text{ cm}} + \frac{1.0}{S'} = 0 \Rightarrow S' = -150.4 \text{ cm} .$$

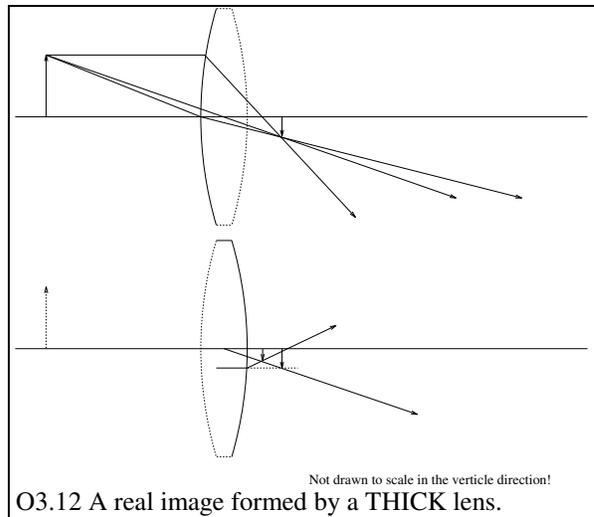
In this case the image is virtual (because of the negative sign of the answer). Here we have again obtained the familiar result that objects under water appear closer to the surface than they really are. The apparent depth, the location of the virtual image, is closer to the surface than the actual object emitting the rays.

The final example, Figure O3.12, includes two surfaces and forms a true thick lens. The object is located 200 cm from a thick piece of glass ($n=1.6$) which has a convex surface with a radius of 30 cm. Our numerical understanding of the behavior of the first surface in this problem is summarized as

$$\frac{1.0}{200 \text{ cm}} + \frac{1.6}{S'} = \frac{1.6 - 1.0}{30 \text{ cm}} \Rightarrow S' = 106.67 \text{ cm} .$$

If the second surface of the thick lens is located 60 cm from the first surface and has a radius of curvature of 30 cm, we are all set to evaluate the behavior there and finish the problem. There are three subtle features that we need to address at the second surface.

The first is that we need to be aware of the fact that the roles of n_1 and n_2 are reversed. Now $n_1=1.6$ and $n_2=1.0$ because the rays are moving out of the glass into the air. The second subtle feature has to do with the object distance that is appropriate in magnitude and sign. The image from the first surface does not fit in the lens and the rays that are approaching the second surface are still converging as they approach the surface.



The magnitude of S is 46.67 cm but the sign is taken to be negative. Had the real image been located ahead of the second surface, in the glass, then we would have a positive value for S . The rays would be diverging as they approached the surface as is normal for an object. Note that virtual images always result in real objects for multiple part systems. With real images as intermediate solutions, we need to exercise some care. Finally, the radius must be taken as negative because the light rays see the curvature of the lens as concave. With those facts in mind we turn to the numerical solution given as

$$\frac{1.6}{-46.67 \text{ cm}} + \frac{1.0}{S'} = \frac{1.0 - 1.6}{-30 \text{ cm}} \Rightarrow S' = 18.4 \text{ cm} .$$

The positive value of S' tells us that the final image, after passing through both surfaces, is real. The rays do, in fact, converge.

The thin lens formula that we used earlier is really based on the ideas presented here with the assumption that no distance separates the two surfaces. However, the graphical techniques based on the idea of a focal point are more insightful. Is there a focal point for refracting surfaces? The answer is obviously yes since we can have parallel rays incident on the interface which will form either a virtual or real image. The concept of “focal point” for refracting surfaces is less useful and more difficult to measure experimentally because often we would need to look for images inside glass. If you really want to know the location of the focal

point, simply take S , the image distance to be infinity, making the $1/S$ term zero, and solve for S' which will be the focal point.

03.6 HOMEWORK PROBLEMS

1. Consider a converging lens with a focal length of 20 cm. If the object is located infinitely far away from the lens so that the rays that are incident on the lens are parallel, find the location of the image.
2. An object that is 1 cm high is located 100 cm from a converging lens with a focal length of 50 cm. Where is the image located and what is the size of the image? Is the image upright or inverted? Is the image real or virtual?
3. Suppose that an object is located at the focal point of a converging lens with a focal length of 30 cm. Where is the image located?
4. An object is located 15 cm from a converging lens with a focal length of 35 cm. Where is the image located and what is the size of the image? Is the image upright or inverted? Is the image real or virtual?
5. Consider a diverging lens with a focal length of -20 cm. If the object is located infinitely far away from the lens so that the rays that are incident on the lens are parallel, find the location of the image.
6. An object that is 1 cm high is located 100 cm from a diverging lens with a focal length of -50 cm. Where is the image located and what is the size of the image? Is the image upright or inverted? Is the image real or virtual?
7. Suppose that an object is located at the focal point of a diverging lens with a focal length of -30 cm. Where is the image located?
8. An object is located 15 cm from a diverging lens with a focal length of -35 cm. Where is the image located and what is the size of the image? Is the image upright or inverted? Is the image real or virtual?
9. Picture a long crown glass with a spherically curved convex end. The radius of curvature is 70 cm. The object is located 100 cm from the interface on the air side of the interface. Where is the image located and what is the size of the image? Is the image upright or inverted? Is the image real or virtual?
10. A thick lens is formed from two spherical surfaces. The first surface encountered by rays emitted from an object is a convex surface of radius 20 cm. The second surface is seen by the light rays to be concave with a radius of -30 cm. The second surface is located 25 cm to the side of the first surface. Where will the image be formed if the object is located 50 cm from the first surface.
11. A thick lens is formed from two spherical surfaces. The first surface encountered by rays emitted from an object is a concave surface -20 cm radius. The second surface is seen by the light rays to be concave with a radius of 30 cm. The second surface is located 25 cm to the side of the first surface. Where will the image be formed if the object is located 50 cm from the first surface.

